Filtering Effect in Waveform Data Inversion for Seismic Source Process

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The Japanese article was machine translated using DeepL. I hope that this article will help you understand the meaning of applying filters when performing waveform inversion.
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§ 1. Introduction

Since the waveform inversion for determining the source process was invented in the 1980s, many researchers have developed analytical methods [e.g., Hartzell and Heaton (1983), Beroza and Spudich (1988), Sekiguchi *et al.* (2000), Ji *et al*. (2002), Yagi and Fukahata (2011)]. Currently, program sources for classical analysis methods are distributed [e.g., Kikuchi and Kanamori (2006)], and these methods have been applied to many earthquakes [e.g., Lay *et al.* (2011)].

In many source process analyses, filters are often applied to the observed waveforms and Green's functions to reduce modeling errors in the Green's functions or to remove noise. However, it is known that the applied filters can change the solution [e.g., Cho and Nakanishi (2000)]. In this section, we discuss the meaning of applying filters in waveform inversion, using a simple case.

§ 2. source process analysis

§ 2.1 Considerations in the time domain

Consider the case of fault slip on a fault plane Σ. For simplicity, the discussion will proceed with the equation for data from a single point and a single component, but the same conclusion can be reached for the case of multiple observation points and multiple components. The observed seismic waveform u can be written as follows.

$$u(t) = \int_{\Sigma} G(\xi, t) * \dot{D}(\xi, t) d\Sigma + e(t). \tag{1}$$

where G and $\dot{D}$ are the Green's functions and the slip rate function on the fault plane Σ , ξ is the coordinates on the fault plane, e is the observation error, and $*$ is the convolution with respect to time. The modeling error is assumed to be negligible. Applying the filter $B(t)$ to the above equation, we obtain

$$B(t) * u(t) = \int_{\Sigma} B(t) * G(\xi, t) * \dot{D}(\xi, t) d\Sigma + B(t) * e(t). \tag{2}$$

The result is as follows. Now, discretizing the slip distribution of the fault in time and space and describing it vectorially, we obtain

$$\mathbf{Bu} = \mathbf{BGm} + \mathbf{Be}. \tag{3}$$

B is the matrix in which the filter is run, an $N \times$ N matrix where *N is the number of* data, **u** is the N-row data vector, **G** is the $N \times M$ kernel matrix where *M is the* number of model parameters, **m is the** M-row model vector, and **e** is the N-row error vector. The least-squares solution $\hat{\mathbf{m}}$ is

$$\hat{\mathbf{m}} = \left(\mathbf{G}^T \mathbf{B}^T \mathbf{C}_d^{-1} \mathbf{BG}\right)^{-1} \mathbf{G}^T \mathbf{B}^T \mathbf{C}_d^{-1} \mathbf{Bu}, \tag{4}$$

where $\mathbf{C}_d$ is the covariance matrix of the filtered data $\mathbf{Bu}$. Assuming that the observation error is uncorrelated and Gaussian with mean zero, the data covariance matrix is

$$\mathbf{C}_d = \sigma^2 \mathbf{BB}^T, \tag{5}$$

where σ^2 is the variance of the observation error. Since the filter matrix is an $N \times$ N square matrix, the least-squares solution is

$$\hat{\mathbf{m}} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{u}, \tag{6}$$

The least-squares solution is obtained independent of the filter employed. In other words, the least-squares solution is obtained independently of the filter employed. This result holds even when constraint conditions are imposed on the solution.

Many analyses treat the data covariance matrix as a diagonal matrix. This assumption corresponds to the assumption that the data covariance matrix after applying the filter can be approximated by a Gaussian distribution with mean zero and no correlation. In other words, the data covariance matrix after applying the filter can be approximated by

$$\mathbf{C}_d = \mathbf{B}\mathbf{e}\mathbf{e}^T\mathbf{B}^T \approx \sigma^2\mathbf{I}, \tag{7}$$

where $\mathbf{I}$ is the identity matrix. If the number of data and the number of model parameters are the same, then $\mathbf{G}$ is a square matrix, and if the inverse matrix exists, then

$$\hat{\mathbf{m}} = \left(\mathbf{G}^T\mathbf{B}^T\mathbf{B}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{B}^T\mathbf{B}\mathbf{u} = \mathbf{G}^{-1}\mathbf{u}. \tag{8}$$

Even when the covariance of the errors is approximated as in equation (7), the result of the analysis is independent of the filter employed.

If the filter is applied only to the observed waveform and not to the Green's function, we obtain a filtered slip time function [Nakano *et al.* (2008)]. The solution in this case is

$$\hat{\mathbf{m}} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{B}\mathbf{u}\,. \tag{9}$$

§ 2.2 Considerations in the frequency domain

To simplify the problem, we assume a point epicenter. The observation equation when the filter is applied at this time is

$$B(t)*u(t) = B(t)*G(\xi_C,t)*\dot{M}_0(t) + B(t)*e(t)\,. \tag{10}$$

The moment-rate function is where ξ_C is the location of the point source and $\dot{M}_0$ is the moment rate function. When discretizing the moment rate function, the number of model parameters and the sampling interval are the same as the number of data and the sampling interval. Fourier transforming equation (10) yields

$$\tilde{B}(f)\tilde{u}(f) = \tilde{B}(f)\tilde{G}(\xi_C,f)\tilde{\dot{M}}_0(f) + \tilde{B}(f)\tilde{e}(f)\,. \tag{11}$$

The following is an example of the Fourier transform. where f is the frequency and the superscript ~ denotes the Fourier transform. Since the moment rate function to be obtained is obtained by division, it can be understood more directly that the solution obtained is independent of frequency.

In actual analysis, it is known that the solution varies depending on the filter applied. Why do the solutions change depending on the filter applied? Many studies employ a condition to constrain the solution by approximating the observation error after applying a filter with an uncorrelated Gaussian distribution with mean zero. Consider the Dumped Least Square Solution (DLSS), which introduces the simplest constraint condition. If the seismic waveforms at times outside the range of analysis are set to zero, the DLSS is obtained by minimizing:

$$\int_{-\infty}^{\infty}\left\{B(t)*\left[G(\xi_C,t)*\dot{M}_0(t)-u(t)\right]\right\}^2 dt+\alpha^2\int_{-\infty}^{\infty}\dot{M}_0^2(t)dt . \qquad (12)$$

where α^2 is a hyperparameter that controls the strength of the constraint. Using Perceval's theorem, eq. (12) becomes

$$\int_{-\infty}^{\infty}\left|\tilde{B}(f)\left[\tilde{G}(\xi_C,f)\tilde{\dot{M}}_0(f)-\tilde{u}(f)\right]\right|^2 df+\alpha^2\int_{-\infty}^{\infty}\left|\tilde{\dot{M}}_0(f)\right|^2 df . \qquad (13)$$

The solution can be obtained independently for each frequency. Therefore, the solution corresponding to the DLSS is

$$\tilde{\dot{M}}_0(f)=\frac{\tilde{B}^*(f)\tilde{B}(f)\tilde{G}^*(\xi_C,f)\tilde{u}(f)}{\tilde{B}^*(f)\tilde{B}(f)\tilde{G}^*(\xi_C,f)\tilde{G}(\xi_C,f)+\alpha^2}, \qquad (14)$$

where the superscript * means complex conjugate. The DLSS solution corresponds to the solution with water level correction [Gubbins (2004)]. In the frequency band removed by the filter, the DLSS solution is more complex than $\left|\tilde{B}(f)\right|\simeq 0$

$$\tilde{\dot{M}}_0(f)\simeq\frac{0}{0+\alpha^2}=0 \qquad (15)$$

Therefore, we obtain a solution that reflects only the DLSS constraint condition that the model weights are zero. In other words, in the frequency band removed by the filter, the solution obtained is determined by the constraint condition. As a result, the analytical results will vary depending on the filter employed. If we return to the time domain, we obtain a solution for the moment rate function with filters similar to those applied to the observed waveform and the Green's function. The same conclusion is obtained when a finite fault model is employed.

Also, the solution corresponding to the DLSS when the observation error before applying the filter is an uncorrelated Gaussian distribution with mean zero is

$$\tilde{\dot{M}}_0(f) = \frac{\tilde{G}^*(\xi_C, f)\tilde{u}(f)}{\tilde{G}^*(\xi_C, f)\tilde{G}(\xi_C, f) + \alpha^2} \tag{16}$$

and the solution obtained is independent of the filter. In other words, the solution changes depending on how the error model is assumed. The setting of the error model is one of the most important items when performing inverse analysis [e.g., Yagi and Fukahata (2011)].

§ 3. Conclusion

The result that the analytical results varied with the filter employed was caused by (1) the assumption that the observation error after applying the filter could be approximated by an uncorrelated Gaussian distribution with mean zero, and (2) the application of constraints.

Considering that the source image in the filtered frequency band is determined by the constraints, it is desirable to use waves with the widest possible bandwidth to obtain an appropriate source image. On the other hand, if a narrow bandwidth filter is applied to the DLSS, the moment rate function will have a filtered source image similar to the filter applied to the observed waveform and the Green's function.

Although not discussed in this paper, further discussion of the properties of the moment-rate functions obtained is warranted if smoothing and nonnegative constraints are applied and a simplified error model is assumed.

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